

Masses of doubly charmed baryons in the extended on-mass-shell renormalization scheme

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In this work, we investigate the mass corrections of the doubly charmed baryons up to N^2LO in the extended-on-mass-shell (EOMS) renormalization scheme, comparing with the results of heavy baryon chiral perturbation theory. We find that the terms from the heavy baryon approach are a subset of those obtained in the EOMS scheme. By fitting the lattice data, we can determine the parameters \bar{m} , α , c_1 and c_7 from the Lagrangian, while in the heavy baryon approach no information on c_1 can be obtained from the baryons mass. Correspondingly, the masses of $m_{\Xi_{cc}^-}$ and $m_{\Omega_{cc}^-}$ are predicted, in the EOMS scheme, extrapolating the results from different values of the charm quark and the pion masses of the lattice QCD calculations.

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I. INTRODUCTION

The doubly charmed baryons are composed of two charmed quarks and one light quark. The ones with quark components ccu , ccd and ccs are named as Ξ_{cc}^{++} , Ξ_{cc}^+ and Ω_{cc}^{++} . Whereas most of their mass comes from their charm content, it is interesting to study the chiral corrections related to the light quark and their influence on the mass splitting.

In the past decades, there has been some experimental effort searching for the doubly charmed baryons [1–4], although the situation is still unsettled. $\Xi_{cc}^+(3520)$ was reported in $\Lambda_c^+ K^- \pi^+$ channel by SELEX collaboration with the mass 3519 ± 1 MeV and the width less than 5 MeV. Later, this state was confirmed in $p D^+ K^-$ channel by SELEX with a mass of 3518 ± 3 MeV. SELEX also have the evidence of the Ξ_{cc}^{++} baryons with masses of 3460 MeV and 3780 MeV which are detected in $\Lambda_c^+ K^- \pi^+ \pi^+$ mode. However, none of these states were confirmed by other experiments, such as FOCUS [5], BABAR [6], Belle [7] and LHCb [8].

Theoretical studies of doubly charmed baryons have been performed with different approaches. Lattice QCD groups predict that the mass of Ξ_{cc} is in the range $3.51 \sim 3.67$ GeV, and that of Ω_{cc}^+ in $3.68 \sim 3.76$ GeV [9–14]. The quark model predictions of Ξ_{cc} and Ω_{cc} masses are in the ranges of $3.48 \sim 3.74$ GeV and $3.59 \sim 3.86$ GeV, respectively [15–28]. In Ref. [29], the mass splitting of doubly charmed baryons is studied in chiral perturbation theory (ChPT) considering heavy diquark symmetry. In Ref. [30], within the framework of heavy baryon ChPT, the mass corrections of doubly charmed baryons were studied up to N^3LO . See Ref. [31] for a review of the current situation on both the theoretical and the experimental side.

Before this latter work, the light baryons' mass corrections had been abundantly studied in ChPT (see reviews [32, 33]); In Refs. [34, 35], the mass corrections of singly heavy baryons were investigated.

Quantum chromodynamics (QCD) is the theory which describes the strong interaction. In the high energy regime, perturbative QCD works very well due to asymptotic freedom, while in the low energy region perturbation theory fails to converge. This low energy region can be studied constructing an effective Lagrangian based on the the QCD symme-

tries and the relevant degrees of freedom. The corresponding theory is ChPT. The Lagrangian is expressed in terms of hadronic fields and organized in the form of a chiral expansion, i.e., an expansion in powers of momentum and light quark masses. When investigating the high order corrections, the chiral power counting scheme, proposed by Weinberg *et al* [36, 37] is used. However, this leads to some difficulties in the baryon sector. Namely, the loop diagrams violate the power counting due to the non-vanishing baryon masses in chiral limit. To solve this issue, various schemes have been proposed, such as heavy baryon chiral perturbation theory [38], infrared baryon chiral perturbation theory [39, 40] and the extended-on-mass-shell (EOMS) approach [41]. See Ref. [42], for a brief explanation and comparison of the three renormalization schemes. The heavy baryon approach was motivated by the methods used in heavy quark effective field theory, in which the baryon is treated to be extremely heavy and acts as a static source. Henceforth, one can take the non relativistic limit and make the expansion in powers of the inverse baryon mass. Within the infrared baryon ChPT, it is used that the infrared singular part of the loop integral conserves the Weinberg's power counting rule. In the EOMS scheme, after calculating the loops covariantly, the power counting breaking terms are subtracted which we will discuss below.

In this work, we will use ChPT with the EOMS renormalization scheme, which has been particularly successful for the light baryon masses [43], to investigate the masses of doubly charmed baryons, and make a comparison with the heavy baryon ChPT results of Ref. [30].

Our work is organized as follows. In Sec. II, the chiral Lagrangian is introduced. We will calculate the doubly charmed baryon masses in the EOMS scheme comparing with the expression from the heavy baryon approach in Sec. III and Sec. IV. Then, we show the numerical results in Sec. V. Finally, a short summary is given.

II. THE EFFECTIVE LAGRANGIAN

In Ref. [30], the effective Lagrangian describing the interaction of doubly charmed baryons and the Goldstone bosons

was constructed. The relevant pieces are

$$\mathcal{L}^{(1)} = \bar{\psi}_0(i\not{D} - m_0 + \frac{g_A}{2}\gamma^\mu\gamma_5 u_\mu)\psi_0, \quad (1)$$

$$\begin{aligned} \mathcal{L}^{(2)} = & c_1 \bar{\psi}_0 \langle \chi_+ \rangle \psi_0 - \left\{ \frac{c_2}{8m_0^2} \bar{\psi}_0 \langle u_\mu u_\nu \rangle \{D^\mu, D^\nu\} \psi_0 + h.c. \right\} \\ & - \left\{ \frac{c_3}{8m_0^2} \bar{\psi}_0 \{u_\mu, u_\nu\} \{D^\mu, D^\nu\} \psi_0 + h.c. \right\} + \frac{c_4}{2} \bar{\psi}_0 \langle u^2 \rangle \psi_0 \\ & + \frac{c_5}{2} \bar{\psi}_0 u^2 \psi_0 + \frac{ic_6}{4} \bar{\psi}_0 \sigma^{\mu\nu} [u_\mu, u_\nu] \psi_0 + c_7 \bar{\psi}_0 \chi_+ \psi_0 \\ & + \frac{c_8}{8m_0} \bar{\psi}_0 \sigma^{\mu\nu} f_{\mu\nu}^+ \psi_0 + \frac{c_9}{8m_0} \bar{\psi}_0 \sigma^{\mu\nu} \langle f_{\mu\nu}^+ \rangle \psi_0. \end{aligned} \quad (2)$$

Note that the fields here are the bare fields and the mass m_0 is the bare mass of the considered doubly heavy baryons. U and u which incorporate the pseudoscalar meson field are defined as

$$U = u^2 = \exp\left(i \frac{\phi_0(x)}{F_0}\right), \quad (3)$$

where $\phi(x)$ is expressed as

$$\phi_0(x) = \begin{pmatrix} \pi_0^0 + \frac{1}{\sqrt{3}}\eta_0 & \sqrt{2}\pi_0^+ & \sqrt{2}K_0^+ \\ \sqrt{2}\pi_0^- & -\pi_0^0 + \frac{1}{\sqrt{3}}\eta_0 & \sqrt{2}K_0^0 \\ \sqrt{2}K_0^- & \sqrt{2}K_0^0 & -\frac{2}{\sqrt{3}}\eta_0 \end{pmatrix}. \quad (4)$$

The doubly heavy baryon field ψ_0 with spin $\frac{1}{2}$ is a column vector in the flavor space, i.e.

$$\psi_0 = \begin{pmatrix} \Xi_{cc}^{++} \\ \Xi_{cc}^+ \\ \Omega_{cc}^+ \end{pmatrix}. \quad (5)$$

In Eq. (1)-(2), $\chi, \chi_\pm, f_{\mu\nu}^R, f_{\mu\nu}^L, f_{\mu\nu}^\pm, u_\mu, \Gamma_\mu, D_\mu$ have the following definition

$$\chi = 2B_0(s + ip), \quad (6)$$

$$\chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u, \quad (7)$$

$$f_{\mu\nu}^R = \partial_\mu r_\nu - \partial_\nu r_\mu - i[r_\mu, r_\nu], \quad (8)$$

$$f_{\mu\nu}^L = \partial_\mu l_\nu - \partial_\nu l_\mu - i[l_\mu, l_\nu], \quad (9)$$

$$f_{\mu\nu}^\pm = u^\dagger f_{\mu\nu}^R u \pm u f_{\mu\nu}^L u^\dagger, \quad (10)$$

$$u_\mu = i[u^\dagger(\partial_\mu - ir_\mu)u - u(\partial_\mu - il_\mu)u^\dagger], \quad (11)$$

$$\Gamma_\mu = \frac{1}{2}[u^\dagger(\partial_\mu - ir_\mu)u + u(\partial_\mu - il_\mu)u^\dagger], \quad (12)$$

$$D_\mu = \partial_\mu + \Gamma_\mu - iv_\mu^{(s)}, \quad (13)$$

where $r_\mu = v_\mu + a_\mu, l_\mu = v_\mu - a_\mu$, and $v_\mu, v_\mu^{(s)}, a_\mu, s, p$ are external c -number fields.

By introducing the renormalized fields through

$$\begin{aligned} \psi &= \frac{\psi_0}{\sqrt{Z_\psi}}, \quad \pi^{\pm,0} = \frac{\pi_0^{\pm,0}}{\sqrt{Z_\pi}}, \quad K^{\pm,0} = \frac{K_0^{\pm,0}}{\sqrt{Z_K}}, \\ \bar{K}^0 &= \frac{\bar{K}_0^0}{\sqrt{Z_K}}, \quad \eta = \frac{\eta_0}{\sqrt{Z_\eta}}, \end{aligned} \quad (14)$$

the Lagrangian of bare fields could be expressed as the sum of basic and counterterm Lagrangians

$$\mathcal{L} = \mathcal{L}_{\text{basic}} + \mathcal{L}_{\text{counterterm}}, \quad (15)$$

where

$$\begin{aligned} \mathcal{L}_{\text{basic}} &= \bar{\psi}_a(i\not{\partial} - m)\psi_a - \bar{\psi}_a \frac{g_A}{2} \gamma_\mu \gamma_5 \partial^\mu \phi_{ab} \psi_b + \dots, \\ \mathcal{L}_{\text{counterterm}} &= (Z_a - 1) \bar{\psi}_a i\not{\partial} \psi_a - (Z_a - 1) \bar{\psi}_a m \psi_a \\ &\quad - Z_a \bar{\psi}_a \delta m_a \psi_a + \dots. \end{aligned} \quad (16)$$

Here, ϕ_{ab} is the renormalized field. The sum is performed for the repeated indices. a and b are the indices in the flavor space ($a, b = 1, 2, 3$ denoting $\Xi_{cc}^{++}, \Xi_{cc}^+, \Omega_{cc}^+$ respectively), m is the mass in the chiral limit, and Z_a is the wave function renormalization constant. Note that here we only show the expression of the lowest order.

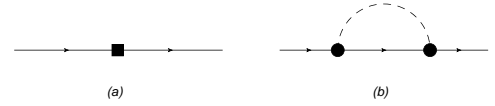


FIG. 1: The Feynman diagrams which contribute to the self-energy of doubly charmed baryon. The solid and dashed lines denote the doubly charmed baryons and Goldstone bosons. The solid dot and black box denote the vertices from the $O(p^1, p^2)$ Lagrangians respectively.

III. THE DOUBLY CHARMED BARYONS MASSES

The full propagator has the form of

$$\begin{aligned} S(p) &= \frac{1}{\not{p} - m_0 - \Sigma(\not{p})} \\ &= \frac{1}{\not{p} - m - \Sigma_r(\not{p})} \\ &\Rightarrow \frac{1}{\not{p} - m - \Sigma_r(\not{p})|_{\not{p}=m_a} - (\not{p} - m_a)[\Sigma_r(\not{p})']|_{\not{p}=m_a}} \\ &= \frac{1}{1 - [\Sigma_r(\not{p})']|_{\not{p}=m_a}} \frac{1}{\not{p} - m_a} \\ &= \frac{Z_a}{\not{p} - m_a}, \end{aligned} \quad (17)$$

where \Rightarrow means the case in the limit of $\not{p} \rightarrow m_a$. In the above function, Z_a is the wave function renormalization constant which is defined as the residue at $\not{p} \rightarrow m_a$, $\Sigma_r(\not{p})$ corresponds to the baryon self-energy, and $m_a = m + \Sigma_r(\not{p})|_{\not{p}=m_a}$ is the physical mass of the baryon. The contributions related to FIG. 1 are

$$\Sigma_a^{(1)} = -2c_1(2m_K^2 + m_\pi^2) - 2c_7[\chi_{aa} - \frac{1}{3}(2m_K^2 + m_\pi^2)], \quad (18)$$

$$\begin{aligned}
\Sigma_{\lambda,ab}^{(2)} &= iC_{ab}^{\lambda} \frac{g_A^2}{4F_{\lambda}^2} \int \frac{d^n k}{(2\pi)^n} \frac{k\gamma_5(\not{p}-\not{k}+m)k\gamma_5}{(p-k)^2-m^2+i\epsilon} \frac{1}{k^2-M_{\lambda}^2+i\epsilon} \\
&= -C_{ab}^{\lambda} \frac{g_A^2}{4F_{\lambda}^2} \left\{ -(p^2-m^2)\not{p} \frac{1}{2p^2} [(p^2-m^2+M_{\lambda}^2)] \right. \\
&\quad \times I_{\lambda a}(-p,0) + I_a(0) - I_{\lambda}(0) + (\not{p}+m)M_{\lambda}^2 \\
&\quad \left. \times I_{\lambda a}(-p,0) + (\not{p}+m)I_a(0) \right\}. \quad (19)
\end{aligned}$$

The index $\lambda = \pi, K, \eta$. M_{λ} and F_{λ} are mass and decay constant of the meson λ , respectively. The coefficient C_{ab}^{λ} is the element of the matrices

$$C^{\pi} = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad C^K = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & 2 \\ 2 & 2 & 0 \end{pmatrix}, \quad C^{\eta} = \begin{pmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{4}{3} \end{pmatrix}. \quad (20)$$

The form of the integral $I_{\lambda,a}$, I_a and I_{λ} are shown in the appendix. Removing the infinite piece in the loop integral using \overline{MS} scheme, we denote the corresponding finite part of the loop integral by $\Sigma_{r,\lambda,ab}^{(2)}$. We extract the term breaking the power counting rule from Eq. (19) as follows

$$\Sigma_r^{break} = C_{ab}^{\lambda} \frac{g_A^2}{32\pi^2 F_{\lambda}^2} \left[-\frac{1}{4m}(p^2-m^2)^2 + mM_{\lambda}^2 \right]. \quad (21)$$

Thus, the mass of the doubly charmed baryons is expressed as

$$\begin{aligned}
m_a &= m + \Sigma_a^{(1)} + \sum_{b=1}^3 \sum_{\lambda=\pi,K,\eta} \Sigma_{r,\lambda,ab}^{(2)}|_{\not{p} \rightarrow m_a} + \delta m_a \\
&\doteq m - 2c_1(2m_K^2 + m_{\pi}^2) - 2c_7 \left[\chi_{aa} - \frac{1}{3}(2m_K^2 + m_{\pi}^2) \right] \\
&\quad + \sum_{b=1}^3 \sum_{\lambda=\pi,K,\eta} (-)C_{ab}^{\lambda} \frac{g_A^2}{4F_{\lambda}^2} 2mM_{\lambda}^2 \frac{1}{(4\pi)^2} \left[-1 + \frac{M_{\lambda}^2}{2m^2} \ln \frac{M_{\lambda}^2}{m^2} \right. \\
&\quad \left. + \frac{M_{\lambda} \sqrt{4m^2 - M_{\lambda}^2}}{m^2} \arccos \frac{M_{\lambda}}{2m} \right] + \delta m_a, \quad (22)
\end{aligned}$$

where δm_a should be the negative of Eq. (21) after substituting $\not{p} = m_a$, i.e.,

$$\delta m_a = - \sum_{b=1}^3 \sum_{\lambda=\pi,K,\eta} C_{ab}^{\lambda} \frac{g_A^2}{32\pi^2 F_{\lambda}^2} m M_{\lambda}^2. \quad (23)$$

And then, we get

$$\begin{aligned}
m_a &= m - 2c_1(2m_K^2 + m_{\pi}^2) - 2c_7 \left[\chi_{aa} - \frac{1}{3}(2m_K^2 + m_{\pi}^2) \right] \\
&\quad + \sum_{b=1}^3 \sum_{\lambda=\pi,K,\eta} (-)C_{ab}^{\lambda} \frac{g_A^2}{4F_{\lambda}^2} 2mM_{\lambda}^2 \frac{1}{(4\pi)^2} \left[\frac{M_{\lambda}^2}{2m^2} \ln \frac{M_{\lambda}^2}{m^2} \right. \\
&\quad \left. + \frac{M_{\lambda} \sqrt{4m^2 - M_{\lambda}^2}}{m^2} \arccos \frac{M_{\lambda}}{2m} \right]. \quad (24)
\end{aligned}$$

Next, we perform the expansion in powers of the Goldstone boson mass

$$\begin{aligned}
m_a &= m - 2c_1(2m_K^2 + m_{\pi}^2) - 2c_7 \left[\chi_{aa} - \frac{1}{3}(2m_K^2 + m_{\pi}^2) \right] \\
&\quad - \sum_{b=1}^3 \sum_{\lambda=\pi,K,\eta} C_{ab}^{\lambda} \frac{g_A^2}{32\pi^2 F_{\lambda}^2} m \left[\frac{\pi M_{\lambda}^3}{m} + \dots \right]. \quad (25)
\end{aligned}$$

Here the ellipsis denotes the contribution of orders higher than 3.

IV. COMPARISON OF DOUBLY HEAVY BARYON MASS IN EOMS AND HEAVY BARYON SCHEMES

Besides the EOMS scheme, heavy baryon chiral perturbation theory (HBChPT) has also been used to study the doubly heavy baryon masses. In this section, we will give a comparison of the results under both schemes. First, we give a brief discussion of HBChPT.

Considering the baryon mass is extremely heavy, we have the picture that the baryon is surrounded by a cloud of light mesons. In this case, the four-momentum of the baryon p^{μ} can be separated into a large piece and a soft residual component

$$p^{\mu} = mv^{\mu} + l^{\mu}, \quad (26)$$

where v^{μ} is the four-velocity, and $v \cdot l \ll m$. Using the projection operator $\mathcal{P}_v^{\pm} \equiv (1 \pm \not{v})/2$, one can define the velocity-dependent fields

$$H = e^{imv \cdot x} \mathcal{P}_v^{+} \psi, \quad h = e^{imv \cdot x} \mathcal{P}_v^{-} \psi, \quad (27)$$

which are also called light and heavy components, respectively. Henceforth, the baryon field ψ is expressed as

$$\psi = e^{-imv \cdot x} (H + h). \quad (28)$$

If projecting the equation of motion onto the \mathcal{P}_v^{+} and \mathcal{P}_v^{-} parts, one has

$$(iv \cdot D + \frac{g_A}{2} \not{v} \gamma_5 + \dots) H + (i\not{D}_{\perp} + \frac{g_A}{2} v \cdot u \gamma_5 + \dots) h = 0, \quad (29)$$

$$(i\not{D}_{\perp} - \frac{g_A}{2} v \cdot u \gamma_5 + \dots) H + (-iv \cdot D - 2m + \frac{g_A}{2} \not{v} \gamma_5 + \dots) h = 0 \quad (30)$$

with $A_{\perp}^{\mu} = A^{\mu} - v \cdot A v^{\mu}$ and the ellipsis means higher order contributions. After solving the Eq. (30) for h and inserting the result into Eq. (29), we arrive at

$$\begin{aligned}
&(iv \cdot D + \frac{g_A}{2} \not{v} \gamma_5 + \dots) H + (i\not{D}_{\perp} + \frac{g_A}{2} v \cdot u \gamma_5 + \dots) \\
&\times (2m + iv \cdot D - \frac{g_A}{2} \not{v} \gamma_5 + \dots)^{-1} (i\not{D}_{\perp} - \frac{g_A}{2} v \cdot u \gamma_5 \\
&+ \dots) H = 0, \quad (31)
\end{aligned}$$

which represents the equation of motion of the field H . Consequently, the corresponding Lagrangian is

$$\mathcal{L}_H = \bar{H} (iv \cdot D + \frac{g_A}{2} \not{v} \gamma_5 + \dots) H + \bar{H} (i\not{D}_{\perp} + \frac{g_A}{2} v \cdot u \gamma_5$$

$$\begin{aligned}
& + \dots)(2m + iv \cdot D - \frac{g_A}{2} \not{u}_\perp \gamma_5 + \dots)^{-1} (i \not{D}_\perp \\
& - \frac{g_A}{2} v \cdot u \gamma_5 + \dots) H
\end{aligned} \quad (32)$$

By expanding the above equation in powers of $1/m$, we obtain

$$\begin{aligned}
\mathcal{L}_H^{(1)} &= \bar{H}(iv \cdot D + g_A S_v \cdot u)H, \\
\mathcal{L}_H^{(2)} &= c_1 \langle \chi_+ \rangle + \frac{c_2}{2} \langle (v \cdot u)^2 \rangle + c_3 \langle v \cdot u \rangle^2 + \frac{c_4}{2} \langle u^2 \rangle \\
&+ \frac{c_5}{2} u^2 + \frac{c_6}{2} [S_v^\mu, S_v^\nu] [u_\mu, u_\nu] + c_7 \hat{\chi}_+ \\
&- \frac{ic_8}{4m} [S_v^\mu, S_v^\nu] f_{\mu\nu}^+ - \frac{ic_9}{4m} [S_v^\mu, S_v^\nu] \langle f_{\mu\nu}^+ \rangle \\
&+ \frac{2}{m} (S_v \cdot D)^2 - \frac{ig_A}{2m} \{S_v \cdot D, v \cdot u\} \\
&- \frac{g_A^2}{8m} (v \cdot u)^2 + \dots
\end{aligned} \quad (33)$$

Using the heavy baryon Lagrangian, one obtains the doubly heavy baryon mass up to chiral order three:

$$\begin{aligned}
m_{Ha} &= m - 2c_1(2m_K^2 + m_\pi^2) - 2c_7(\chi_{aa} - \frac{1}{3}(2m_K^2 + m_\pi^2)) \\
&- \sum_{b=1}^3 \sum_{\lambda=\pi, K, \eta} C_{ab}^\lambda \frac{g_A^2}{(4\pi F_\lambda)^2} \frac{\pi}{2} M_\lambda^3.
\end{aligned} \quad (34)$$

This expression coincides exactly with Eq. (25). However, Eq. (25) is just a truncated Taylor expansion of the full order three EOMS result (Eq. (24)). The EOMS result, automatically includes higher orders from the loop calculations, like the logarithmic and the arccosinus terms of Eq. (24). These terms, apart from reflecting the proper analytic dependence coming from the loops, have proved to lead to a faster chiral convergence in many cases [42–50].

V. NUMERICAL RESULTS

In Eq. (24), there are three low energy constants c_1 , c_7 and g_A . As mentioned in Ref. [30], g_A can be fixed by comparing with other theoretical calculations. In Ref. [51], the Lagrangian depicting doubly heavy baryon and meson interaction is constructed based on the heavy diquark symmetry

$$\mathcal{L} = \text{Tr}[T_a^\dagger (iD_0)_{ba} T_b] - g \text{Tr}[T_a^\dagger T_b \vec{\sigma} \cdot \vec{A}_{ba}] + \dots, \quad (35)$$

where $T_{a,i\beta} = \sqrt{2}(\Xi_{a,i\beta}^* + \frac{1}{\sqrt{3}}\Xi_{a,\gamma}\sigma_{\gamma\beta}^i)$. By fitting the D^{*+} decay width, one gets the coupling $g = 0.6$. Comparing the Lagrangians with ours, we obtain $g_A = -g/3 = -0.2$.

Besides the doubly heavy baryon mass m in chiral limit, the low energy constants c_1 and c_7 still need to be determined. In order to do this, we fit the lattice data in Ref. [13].

The masses of Ξ_{cc} are given for different m_π and m_c in Ref. [13]. The strange quark mass is tuned as to reproduced the kaon mass. As in Ref. [30], we assume that only the bare mass m depends on the valence charm quark mass m_c and use the same ansatz as in Refs. [13, 30]

$$m = \tilde{m} + 2m_c + \alpha/m_c + \mathcal{O}(1/m_c^2). \quad (36)$$

TABLE I: Values and uncertainties of the parameters \tilde{m} , α , c_1 and c_7 obtained by fitting the lattice data from [13].

	\tilde{m} (GeV)	α (GeV ²)	c_1	c_7	$\chi_{d.o.f}^2$
value	3.110	-0.459	-0.098	-0.073	
error	0.111	0.047	0.045	0.089	0.22

TABLE II: Masses of Ξ_{cc} and Ω_{cc} and their corresponding uncertainties for the different values of the charm quark mass from lattice QCD. All the values are expressed in GeV.

m_c^{phy}	$m_{\Xi_{cc}}$	$m_{\Omega_{cc}}$
0.598 ± 0.066	3.608 ± 0.218	3.663 ± 0.223
0.591 ± 0.028	3.585 ± 0.166	3.640 ± 0.173
0.598 ± 0.070	3.608 ± 0.225	3.663 ± 0.230

The physical mass m_c^{phy} is tuned to reproduce the mass of the D meson at the physical point in Ref. [13]. And here, for the different lattice parameters $\beta = 3.9$, $\beta = 4.05$ and $\beta = 4.2$, the values of physical charm quark mass m_c^{phy} are given as 0.598 ± 0.066 , 0.591 ± 0.028 and 0.598 ± 0.070 GeV, respectively. The results of our fit to the lattice data are shown in Table. I.

In Fig. 2, we plot the best fit results. The doubly heavy baryons masses are shown in Table. II for the different values of the physical charm quark mass obtained by lattice QCD. Finally, the averages for these masses are $m_{\Xi_{cc}} = 3.597 \pm 0.114$ MeV and $m_{\Omega_{cc}} = 3.652 \pm 0.118$ MeV.

Our results, which are obtained by a chiral extrapolation from lattice data, although consistent with the SELEX measurement ($m_{\Xi_{cc}} = 3519 \pm 1$ MeV) because of their large error bars, agree better with most theoretical estimates predicting a larger mass for this baryon. In Table VIII of Ref. [28], a wide compilation of theoretical predictions can be found.

In Ref. [30], using the heavy baryon approach, the same set of parameters is also determined by fitting the lattice data from [13]. However, in this latter approach, one can not give any information of the low energy constant c_1 , since the term corresponding to c_1 is the same constant for all cases and the term coming from the loop contribution does not depend on the baryon mass (so that the term corresponding to c_1 can be absorbed into the baryon mass in the chiral limit).

VI. SUMMARY

The doubly heavy baryons are very interesting hadronic systems although the experimental situation needs still to be settled. The charmed quarks are relatively heavy so that they can be treated as spectators. Consequently, the chiral dynamics is solely governed by the light quark. In this work, we have used an effective Lagrangian which describes the chiral dynamics of doubly heavy baryons to calculate chiral corrections to their masses. In order to deal with the power counting problem, intrinsic to baryon ChPT, we have used the EOMS method. Within this scheme, we have obtained the baryon masses up to N²LO. We have shown that a truncation of our results reproduces those of the heavy baryon approach.

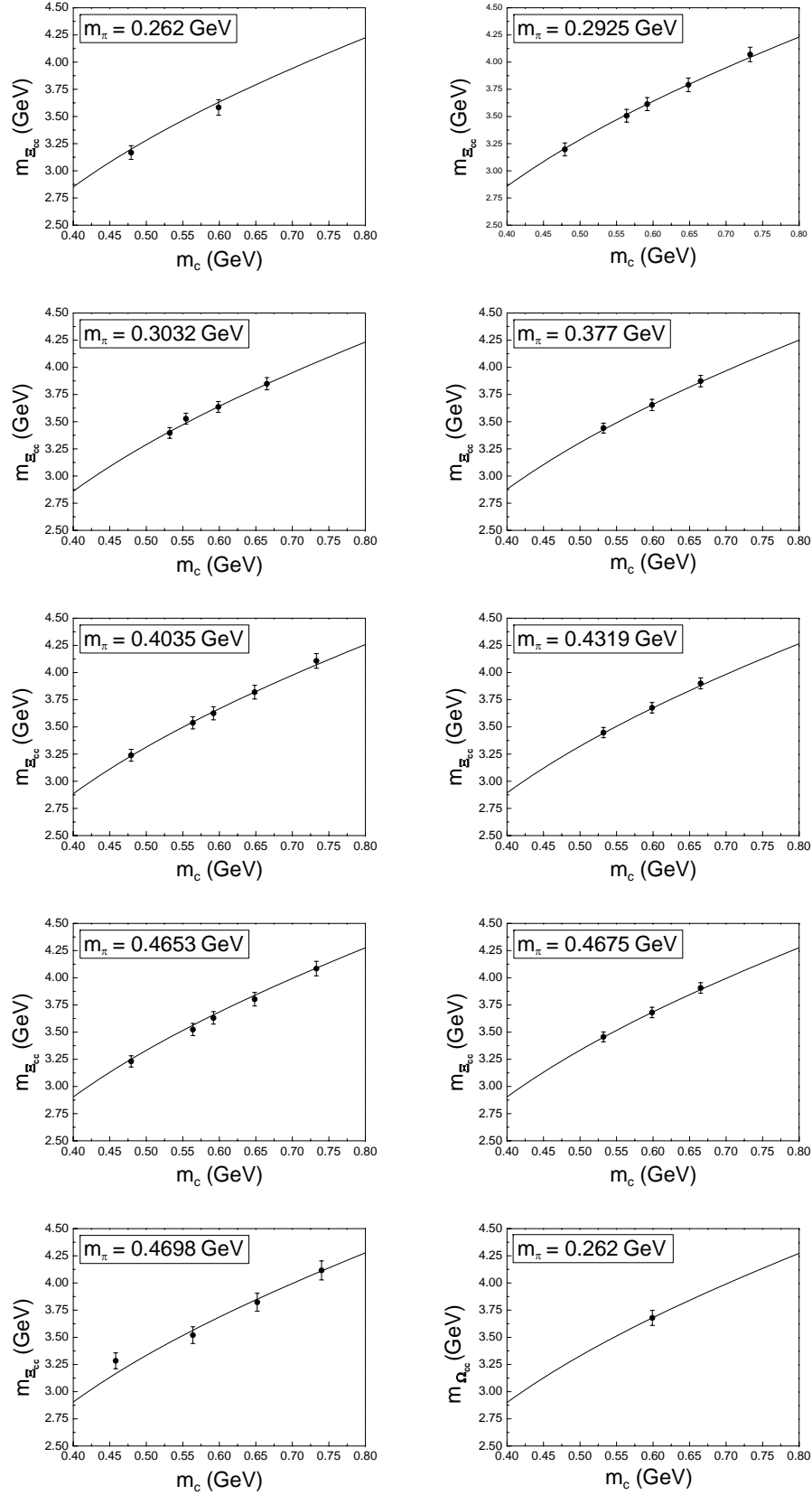


FIG. 2: Masses of Ξ_{cc} and Ω_{cc} as a function of m_c for different pion masses. The dots are lattice data from Ref. [13], and the solid curves are our fitted result in the EOMS renormalization scheme.

We have also performed a numerical analysis of the doubly heavy baryon masses. From the D^{*+} decay width one gets the coupling $g_A = -0.2$. Then, by fitting the lattice data, at several pion and charm quark masses, from Ref. [13], the parameters c_1 , c_7 , \bar{m} and α have been determined. Consequently, the masses of the doubly charmed baryons at the physical point have been predicted. The use of the EOMS scheme has allowed us to determine the constant c_1 which in the heavy baryon method is fully correlated with the baryon mass in the chiral limit and cannot be disentangled. We expect that, as in many other observables, the chiral convergence of EOMS will be faster than in the heavy baryon approach. We are looking forward to more developments of experimental and theoretical studies in this field, through which we can deepen our understanding of the hadron spectrum and nonperturbative QCD.

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Appendix: Loop integrals

In this appendix, we give the loop integrals which are needed in our calculation.

$$I_a(0) = i \int \frac{d^n k}{(2\pi)^n} \frac{1}{k^2 - m^2 + i\epsilon} = 2m^2 \bar{\lambda},$$

$$\begin{aligned} I_\lambda(0) &= i \int \frac{d^n k}{(2\pi)^n} \frac{1}{k^2 - M_\lambda^2 + i\epsilon} \\ &= 2M_\lambda^2 \bar{\lambda} + \frac{M_\lambda^2}{(4\pi)^2} \ln \frac{M_\lambda^2}{m^2}, \end{aligned}$$

$$\begin{aligned} I_{\lambda a}(-p, 0) &= i \int \frac{d^n k}{(2\pi)^n} \frac{1}{(p-k)^2 - m^2 + i\epsilon} \frac{1}{k^2 - M_\lambda^2 + i\epsilon} \\ &= 2\bar{\lambda} + \frac{1}{(4\pi)^2} \left(-1 + \frac{p^2 - m^2 + M_\lambda^2}{2p^2} \ln \frac{M_\lambda^2}{m^2} + f_0 \right), \end{aligned}$$

where

$$\bar{\lambda} = \frac{1}{(4\pi)^2} \left[-\frac{1}{\epsilon} - \frac{1}{2} \left(\ln \frac{4\pi}{m^2} + 1 - \gamma \right) \right],$$

$$f_0 = \begin{cases} \frac{\sqrt{\Theta^2 - \Delta^2}}{p^2} \arccos \frac{-\Delta}{\Theta}, & -1 < \frac{\Delta}{\Theta} < 1 \\ \frac{\sqrt{\Delta^2 - \Theta^2}}{2p^2} \ln \frac{\Delta + \sqrt{\Delta^2 - \Theta^2}}{\Delta - \sqrt{\Delta^2 - \Theta^2}}, & \frac{\Delta}{\Theta} < -1 \\ \frac{\sqrt{\Delta^2 - \Theta^2}}{2p^2} \ln \frac{\Delta + \sqrt{\Delta^2 - \Theta^2}}{\Delta - \sqrt{\Delta^2 - \Theta^2}} - i\pi \frac{\sqrt{\Delta^2 - \Theta^2}}{p^2}, & \frac{\Delta}{\Theta} > 1 \\ 0, & \frac{\Delta}{\Theta} = \pm 1 \end{cases}$$

with $\Delta = p^2 - m^2 - M_\lambda^2$ and $\Theta = 2mM_\lambda$.

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